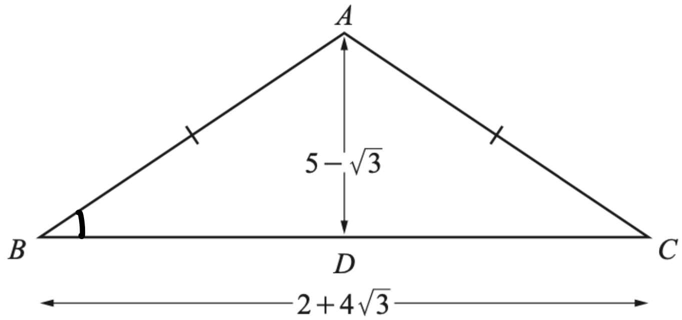
Chapter 3 Indices and Surds

1. **DO NOT USE A CALCULATOR IN THIS QUESTION.**

   In this question all lengths are in centimetres.



   The diagram shows the isosceles triangle *ABC*, where *AB* = *AC* and *BC* = $2 + 4\sqrt{3}$. The height, *AD,* of the triangle is $5 - \sqrt{3}$.

   a. Find the area of the triangle *ABC,* giving your answer in the form $a + b\sqrt{3}$, where a and b are integers. [2]

   $A = \frac{1}{2}bh$
   $= \frac{1}{2} \times 2 + 4\sqrt{3} \times 5 - \sqrt{3}$
   $= (1 + 2\sqrt{3}) \times 5 - \sqrt{3}$
   $= 5 - \sqrt{3} + 10\sqrt{3} - 6 = 9\sqrt{3} - 1$

   b. Find $\tan ABC$, giving your answer in the form $c + d\sqrt{3}$, where c and d are integers. [3]

   $\tan B = \frac{5 - \sqrt{3}}{1 + 2\sqrt{3}} \times \frac{1 - 2\sqrt{3}}{1 - 2\sqrt{3}}$
   $= \frac{5 - 10\sqrt{3} - \sqrt{3} + 6}{1 - 12}$
   $= \frac{11 - 11\sqrt{3}}{-11} = \sqrt{3} - 1$

   c. Find $sec^2 ABC$, giving your answer in the form $e + f\sqrt{3}$, where e and f are integers. [2]

   $\sec^2 ABC = 1 + \tan^2 ABC$
   $= 1 + (\sqrt{3} - 1)^2$
   $= 1 + 3 - 2\sqrt{3} + 1$
   $= 5 - 2\sqrt{3}$

2. **DO NOT USE A CALCULATOR IN THIS QUESTION.**

Find the positive solution of the equation $(5 + 4\sqrt{7})x^2 + (4 - 2\sqrt{7})x - 1 = 0$, giving your answer in the form $a + b\sqrt{7}$, where *a* and *b* are fractions in their simplest form.

[5]

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 + 2\sqrt{7} \pm \sqrt{(4 - 2\sqrt{7})^2 - 4(5 + 4\sqrt{7})(-1)}}{2(5 + 4\sqrt{7})}$$

$$= \frac{-4 + 2\sqrt{7} \pm \sqrt{16 - \cancel{16\sqrt{7}} + 28 + 20 + \cancel{16\sqrt{7}}}}{2(5 + 4\sqrt{7})}$$

$$= \frac{-4 + 2\sqrt{7} \pm \sqrt{64}}{2(5 + 4\sqrt{7})}$$

$$= \frac{-4 + 2\sqrt{7} \pm 8}{2(5 + 4\sqrt{7})} = \frac{-4 + 2\sqrt{7} + 8}{2(5 + 4\sqrt{7})} \text{ or } \frac{-4 + 2\sqrt{7} - 8}{2(5 + 4\sqrt{7})}$$

$$= \frac{4 + 2\sqrt{7}}{2(5 + 4\sqrt{7})} \text{ or } \frac{-12 + 2\sqrt{7}}{2(5 + 4\sqrt{7})}$$

$$= \frac{2 + \sqrt{7} \times 5 - 4\sqrt{7}}{5 + 4\sqrt{7} \times 5 - 4\sqrt{7}} \qquad \frac{-6 + \sqrt{7} \times (5 - 4\sqrt{7})}{5 + 4\sqrt{7} \times (5 - 4\sqrt{7})}$$

$$= \frac{10 - 8\sqrt{7} + 5\sqrt{7} - 28}{25 - 112} \text{ or } \frac{-30 + 24\sqrt{7} + 5\sqrt{7} - 28}{-87}$$

$$= \frac{-18 - 3\sqrt{7}}{-87} \qquad \frac{-58 + 29\sqrt{7}}{-87} \div 29$$

$$= \frac{6 + \sqrt{7}}{29} \qquad = \frac{2 - \sqrt{7}}{3}$$

$$= \frac{6}{29} + \frac{\sqrt{7}}{29} \qquad = \frac{2}{3} - \frac{\sqrt{7}}{3}$$

(reject)

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3. **DO NOT USE A CALCULATOR IN THIS QUESTION.**

The point $(1 - \sqrt{5}, p)$ lies on the curve $y = \frac{10+2\sqrt{5}}{x^2}$. Find the exact value of *p*, simplifying your answer.

[5]

$$p = \frac{10+2\sqrt{5}}{(1-\sqrt{5})^2}$$

$$= \frac{10+2\sqrt{5}}{1-2\sqrt{5}+5}$$

$$= \frac{10+2\sqrt{5}}{6-2\sqrt{5}}$$

$$= \frac{5+\sqrt{5}}{3-\sqrt{5}} \quad \begin{matrix} \times\, 3+\sqrt{5} \\ \times\, 3+\sqrt{5} \end{matrix}$$

$$= \frac{15+5\sqrt{5}+3\sqrt{5}+5}{9-5}$$

$$= \frac{20+8\sqrt{5}}{4} = 5+2\sqrt{5}$$

4. Solve the equation $\frac{9^{5x}}{27^{x-2}} = 243.$

[3]

$$\frac{(3^2)^{5x}}{3^{3x-6}} = 3^5$$

$$3^{10x-3x+6} = 3^5$$

$$7x+6 = 5$$

$$7x = -1$$

$$x = -\frac{1}{7}$$

5. **DO NOT USE A CALCULATOR IN THIS QUESTION.**

   a. Simplify $\frac{\sqrt{128}}{\sqrt{72}}$.

$$= \frac{\sqrt{64 \times 2}}{\sqrt{36 \times 2}} = \frac{8\cancel{\sqrt{2}}}{6\cancel{\sqrt{2}}} = \frac{4}{3}$$

[2]

   b. Simplify $\frac{1}{1+\sqrt{3}} - \frac{\sqrt{3}}{3+2\sqrt{3}}$, giving your answer as a fraction with an integer denominator.

$$\frac{3+2\sqrt{3}-\sqrt{3}(1+\sqrt{3})}{(1+\sqrt{3})(3+2\sqrt{3})}$$

[4]

$$= \frac{\cancel{3}+2\sqrt{3}-\sqrt{3}-\cancel{3}}{3+2\sqrt{3}+3\sqrt{3}+6} = \frac{\sqrt{3}}{9+5\sqrt{3}} \times \frac{9-5\sqrt{3}}{9-5\sqrt{3}}$$

$$= \frac{9\sqrt{3}-15}{81-75} = \frac{9\sqrt{3}-15}{6} = \frac{3\sqrt{3}-5}{2}$$

6. Write $\frac{\sqrt{p}(qr^2)^{\frac{1}{3}}}{(q^3p)^{-1}r^3}$ in the form $p^a q^b r^c$, where a, b and c are constants.

$$\frac{p^{1/2}\, q^{1/3}\, r^{2/3}}{q^{-3}\, p^{-1}\, r^{3}} = p^{3/2}\, q^{10/3}\, r^{-7/3}$$

[3]

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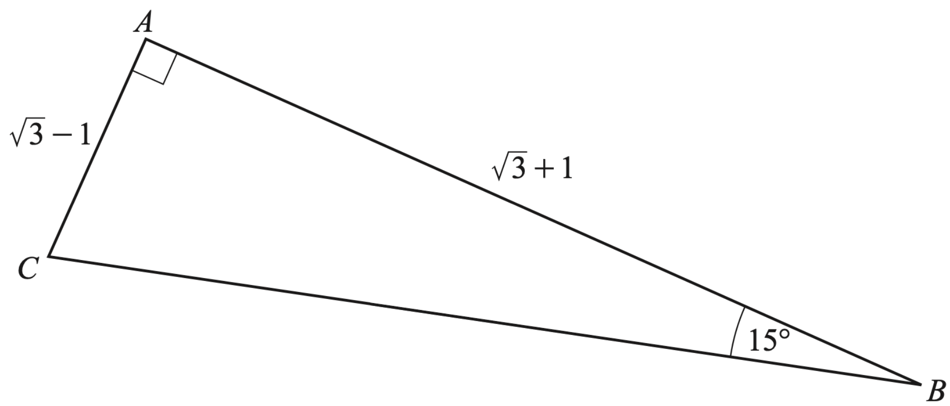

In the diagram above, $AC = \sqrt{3} - 1$, $AB = \sqrt{3} + 1$, angle $ABC = 15°$ and angle $CAB = 90°$.

a. Show that $tan\ 15° = 2 - \sqrt{3}$.

[3]

$$\tan 15 = \frac{AC}{AB}$$

$$= \frac{\sqrt{3}-1 \times \sqrt{3}-1}{\sqrt{3}+1 \times \sqrt{3}-1}$$

$$= \frac{3-2\sqrt{3}+1}{3-1} = \frac{4-2\sqrt{3}}{2} = 2-\sqrt{3}$$

(shown)

b. Find the exact length of BC.

[2]

$$BC^2 = (\sqrt{3}-1)^2 + (\sqrt{3}+1)^2$$

$$= 3-\cancel{2\sqrt{3}}+1+3+\cancel{2\sqrt{3}}+1$$

$$= 8$$

$$BC = 2\sqrt{2}$$

8. Find the value of x such that $\frac{4^{x+1}}{2^{x-1}} = 32^{\frac{x}{3}} \times 8^{\frac{1}{3}}$.

[4]

$$\frac{2^{2x+2}}{2^{x-1}} = 2^{5x/3} \times 2^1$$

$$2^{x+3} = 2^{\frac{5x}{3}+1}$$

$$x+3 = \frac{5x}{3} + 1$$

$$2 = \frac{2x}{3}$$

$$\frac{6}{2} = x$$

$$x = 3$$

9. Solve the following simultaneous equations.

$$3^x \times 9^{y-1} = 243$$

$$8 \times 2^{y-\frac{1}{2}} = \frac{2^{2x+1}}{4\sqrt{2}}$$

[5]

$$3^x \times 3^{2y-2} = 3^5 \longrightarrow x+2y-2 = 5$$

$$x + 2\cancel{y} = 7 \text{ — } ①$$

$$4x - \cancel{2y} = 8 \text{ — } ②$$

$$5x = 15$$

$$x = 3$$

$$2y = 7-3$$

$$= 4$$

$$y = 2$$

$$2^3 \times 2^{y-\frac{1}{2}} = \frac{2^{2x+1}}{2^{2\frac{1}{2}}}$$

$$3+y-\frac{1}{2} = 2x-\frac{3}{2}$$

$$6+2y-1 = 4x-3$$

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